SAGE Virtual Summer Camp | August 1-7, 2021

Spectroscopy and Stellar Physics

An Introduction to the Thermal Blackbody

1 Introduction: What are spectra?

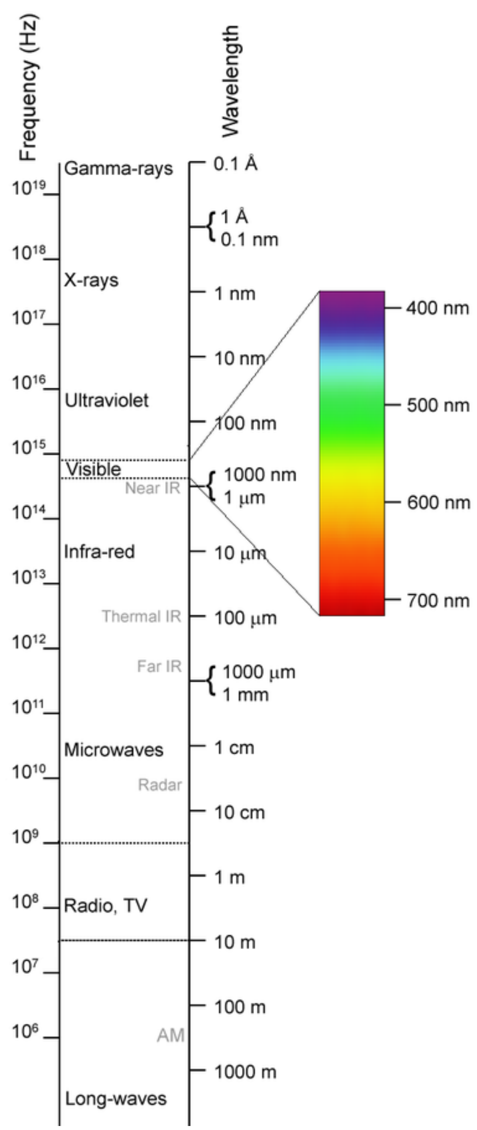

Figure 1: The electromagnetic spectrum.

A **spectrum** is a measure of intensity of radiation across many wavelengths. Electromagnetic (EM) radiation, or light, is an indispensable probe of physics for far-off systems like stars, extrasolar planets, and even other galaxies. We frequently measure spectra to understand what's happening in these distant places. The domain of visible light is a small part of the electromagnetic spectrum, as you can see on the left.

The energy of the light we see coming from the object and its temperature are linked - you can imagine that the hotter an object is, the more energetic the photons it radiates are. A **blackbody** is considered a perfect radiator, something that is emitting photons, and has a well understood spectrum based upon the underlying physics (characterized by the *Planck function* seen in figure 2). Real objects tend to be imperfect radiators, because energy is often lost to other processes like scattering, kinetic motion, or thermal expansion. Some hot, astrophysical objects like stars share a lot of similarities with our perfect blackbody radiator. Stars also tend to have fingerprints of their composition overlaid onto this continuous blackbody template, emission and absorption features that can tell us about what sort of metals are present.

1.1 Wien's Law

Wien's displacement law links the peak wavelength, λ_p of a blackbody spectrum to its temperature, T. This peak is where the most light is emitted.

$$\lambda_p = \frac{b}{T}$$

where b is a constant equal to $2.897 \times 10^{-3} Km$ (Kelvin-meters). This means we can look at a spectrum of things similar to blackbodies and determine the effective temperature of the surface of the object we are looking at!

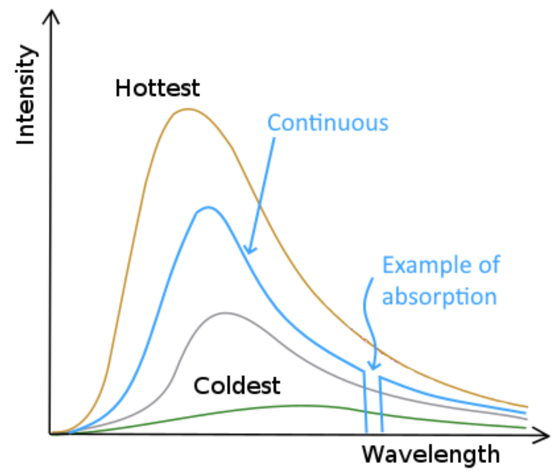

Figure 2: A depiction of the Planck blackbody spectrum. The peak wavelength gets lower as temperature goes up. Imperfect radiators might have absorption features as seen in the blue line.

1.2 What can you use to see spectra?

You have been supplied a simple spectroscope that will let you split visible light along the wavelength axis. This can be done with *diffraction* through a single slit, through many slits in a *diffraction grating*, or with refraction through a prism. You have a single slit spectroscope, if you align the slit with a particular source and look through the viewing port on the opposing side, the spectrum of the source in the visible range is projected along the printed axis.

2 Examining Common Light Sources

2.1 Lightbulbs

1. Find the nearest lightbulb and examine the spectrum with your spectroscope.

2. Describe what you see. What wavelengths of light are present? Is the spectrum continuous, or are there breaks?
   Solution:
   If you have an incandescent lightbulb, the spectrum will be continuous across the entire visible range, peaking in the yellow. Phosphorescent lightbulbs will have distinct emission regions (breaks).

3. If all of the light emitted from the lightbulb was in the optical domain, how might we find the minimum total amount of energy it needs to be powered? (Hint: think about what measurements could be made)
   Solution:
   The total energy is given by adding up all the light emitted at these different wavelengths, taking into account energy at each frequency. This would be the integral of a curve like that in Fig. 2. We therefore have to know both the wavelength and intensity.

4. Alternatively, if we know the lightbulb requires 60 Watts (Joules/second, a measurement of *power*), but we only see the equivalent of 30 Watts in our spectroscope, where might the extra energy be going?
   Solution:
   Since our spectroscope only looks at the visible wavelengths (400-800 nm), it's possible 30 Watts of energy are emitted in the infrared / released as heat. Additionally, it's possible that the lightbulb is electronically inefficient, and some of that energy is released as heat in places besides the bulb.

5. Estimate where the peak of this spectrum is (which wavelength looks brightest on the axis?). Use this peak to estimate the corresponding blackbody temperature of the lightbulb with Wien's law. (Hint: a nanometer is $10^{-9}m$)
   Solution:
   To my eye, with an incandescent bulb, the spectrum looks brightest at 600 nm. Using Wein's law:

$$T = b/\lambda_p = 2.897 \times 10^{-3} Km / 6 \times 10^{-7} m = 4,828\ K \quad or \quad 4,554\ C$$

6. Is your answer reasonable? Is the lightbulb a good example of a blackbody radiator?
   Solution:
   No! It is entirely unreasonable that a lightbulb in a household would be thousands of degrees. I can touch it without burning myself, so odds are the true temperature is closer to 50 C. This means that a radiating blackbody is a bad physical model for a lightbulb.

2.2 LEDs

1. Point your spectroscope at the nearest electronic screen (this could be a laptop, television, or phone) and examine the spectrum. Note: this works best with bright sources, and is easiest to see when the screen in question is white.

2. Describe what you see. What wavelengths of light are present? Is the spectrum continuous, or are there breaks?
   Solution:
   The white light seems to cover most of the visible range, but there are distinct breaks. There appears to be clear band separation between red, green, and blue.

3. Given what you see, do you expect this emitter to spend more, equal, or less power emitting than the lightbulb? Why? What information could change your answer if you had a different measurement device? (Hint: think about both intensity of light and your answer to the previous question)
   Solution:
   Since there are gaps in the spectrum, I would expect the LED to supply less total power than an incandescent lightbulb. This is only the case if the total intensities are comparable - that is, if we look at a LED screen that is much much brighter overall than a lightbulb, the screen would still be eating more power. Another aspect that could change your answer (besides an axis with specified intensity), would be more wavelength information. Perhaps the LED emits a lot of light in the infrared, where the lightbulb emits only a little. We can make an educated guess that this isn't the case, because an LED is cool to the touch and a lightbulb is comparably hotter - and therefore is emitting more heat/infrared light than the LED.

4. How does the spectrum change if you look at only a red part of the screen (rather than white)? How many kinds of LEDs do you think are in your screen?
   Solution:
   We see the red band isolated from the rest of the spectrum. We could do a similar exercise with a green and blue part of the screen to find that we can isolate three separate LED types, that are the standard RGB ensemble.

3 What can we learn about the Sun?

Since we know the form of the blackbody spectrum, and that the form depends only on temperature, we might guess that the total energy emitted by an blackbody also only depends on the temperature. This is described in the *Stefan-Boltzmann law*:

$$P = A\,\sigma_{sf}\,T^4$$

where P is the total power (energy/time) emitted by the blackbody across *all wavelengths*, T is the temperature, A is the surface area, and σ_{sf} is the Stefan-Boltzmann constant, which is $5.67 \times 10^{-8} W m^{-2} K^{-4}$ With this equation, we have the powerful ability to look at an object, get its temperature, and use that temperature to estimate how much energy is needed to keep that object going. With stars, this is particularly powerful since they are very similar to blackbody emitters.

1. Step outside, or point your spectroscope out a window at either the sky or a piece of white paper that reflects the sun's light back at you. (Note: Do NOT look directly at the sun, even through your spectroscope)

2. Describe what you see. Is the spectrum continuous?
   Solution:
   Yes, the spectrum seems to be continuous, and is brightest in the yellow/green domain. If you've got a really good eye, you might notice a few "breaksßimilar to the LED scenario. The most noticable one to me is at 590 nm, which corresponds to sodium content in the solar photosphere. We can use features like this to identify some of the kinds of gases and metals in stars, which informs our understanding of stellar lifecycles and fusion.

3. Estimate where the peak of this spectrum is (which wavelength looks brightest on the axis?). Use this peak to estimate the corresponding blackbody temperature of the sun with Wien's law. (Hint: a nanometer is $10^{-9} m$) If the sky is overcast, give your best guess on the peak wavelength using your knowledge of color and Fig. 1.
   Solution:
   My eye sees the brightest light to be somewhere in the green domain, around 500 nm, though finding the peak by eye this way is difficult, and astronomers usually have an axis devoted to brightness! This gives us a blackbody temperature estimate of

$$T = b/\lambda_p = 2.897 \times 10^{-3} Km/5.0 \times 10^{-7} m = 5,700\ K \ \ or \ \ 5,427\ C$$

4. Is your answer reasonable? Is the sun a good example of a blackbody radiator? (Hint: look up the surface temperature of the sun and compare your answer)
   Solution:
   Yes! In fact, we're only tens of degrees off with this estimate from the actual effective temperature of the solar photosphere, which is typically 5,500 C.

5. It turns out that what you measure is the effective temperature of the *stellar photosphere*, or the outer atmosphere. If we measure the sun to have a radius of $6.957 \times 10^{8} m$ and the temperature

you calculated, what is its power output? (Hint: Use the Stefan-Boltzmann law)
Solution:
Using the Stefan-Boltzmann law, we can see that:

$$P = A\,\sigma_{sf}\,T^4 = 4\pi \times (6.957 \times 10^8 m)^2 \times 5.67 \times 10^{-8} W m^{-2} K^{-4} \times (5,700K)^4 = 3.64 \times 10^{26} W$$

6. The sun's radiation comes from nuclear fusion, converting a small fraction of mass to energy (around 0.7% of the core). We can measure the solar mass by looking at how its gravity affects us and other planets. If we know the mass, and the physics of proton-proton fusion, we can estimate that the sun has 1.2×10^{44} Joules to burn into light. Use the relation between energy, power, and time to estimate the lifetime of the sun using your previous answer. (Hint: A Watt is 1 Joule per second).
Solution:
We know that power is energy per unit time, so we can estimate the time our sun has if it burns at constant power using the following:

$$P = E/t, \;\; t = E/P = 1.2 \times 10^{44} J/3.64 \times 10^{26} W = 3.3 \times 10^{17} s = 10.4 \times 10^9\; years$$

7. The Earth is 4.5 billion years old, and the universe approximately 13.8 billion years old. How does the sun's remaining lifespan compare?
Solution:
10 billion years is the same order of magnitude as the age of the universe! Our sun has a long life ahead of it (what astronomers call its *main sequence lifetime*), after which it can perform different kinds of fusion in its core and will have to find a new hydrostatic equilibrium. What this means for us is that the sun will expand to be so large that Earth's orbit will lie within the solar radius! Fortunately, this won't happen for billions of years.

Using a simple piece of plastic and stellar physics you've made a concrete prediction about the future of our solar system! This exercise has hopefully shown how powerful light is as a tool to learn about the universe around us, and has taught you a little about spectroscopy. Ask your lab volunteer for the solutions to check your work!